Hydraulic performance of siphonic turbine in low head sites

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Abstract

A suggested approach to low head hydropower at run-of-river dams and tidal estuary sites is to include an intermediate air transmission stage where water pressure is converted into air pressure. Several studies have been carried out to investigate the hydraulic performance of such system called siphonic turbine. However, their overoptimistic assessment of the performance of the system is not applicable to actual low head sites.

This paper investigates the performance of a siphonic turbine considered to be installed in Grevelingen Brouwersdam in the Netherlands. Based on available experimental results on air–water ow in downward sloping pipes, a one-dimensional numerical hydrodynamic model is developed to nd optimised combinations of air and water ow rates. The generated power of an air turbine, connected to the siphon, is calculated and the efficiency is determined for several hydraulic and geometrical conditions. A maximum efficiency of 7.2% is observed, which is way below the estimated values in previous studies. The total generated power is about 4 MW for a total water discharge of 4500 m^3/s and an available head of 1.25 m. This relatively low performance has to be kept in mind when identifying the best available technology for low head sites.

1. Introduction

Hydropower accounts for more electricity production than solar PV, wind, and geothermal combined [1]. In 2012, hydropower accounted for 16.5% of the worlds electricity production [2]. As a form of hydropower, tidal energy has traditionally suffered from relatively high cost and been limited to the sites with suf ciently high tidal ranges or ow velocities that constrict its total availability. However, various technologies have been developed to overcome these constrictions and to use this broad source of energy (Wiemann et al. [3]), namely pressure turbines (e.g. bulb and Kaplan turbines) and free-steam turbines (e.g. Davis and KHPS turbines). In all of these technologies, the electrical energy is generated by a water turbine.

Recently, siphonic turbine has been studied as an environment friendly solution for low head sites (Bellamy [4], French and Widden [5]). In a siphonic turbine, power is produced by the admission of air into a siphon. The general con guration of a siphonic turbine is shown in Fig. 1. An air inlet is located on an elevated part of the siphon where the pressure is sub-atmospheric. Due to pressure difference, ambient air ow into the siphon through an air turbine. The turbine is connected to a generator which produces electrical energy. Comparing to the other technologies, the siphonic turbine has several advantages such as easy acceptability for maintenance, being inherently fail-safe and environmental bene ts, e.g. aeration of water and fish friendliness.

The rst experiment to produce electricity by siphonic turbine was carried out by Bellamy [4]. He developed a simple mathematical model which predicted an efficiency of 50%. Moreover, a prototype was built and tested with a 2.3 m of head which produced 22 kW energy, achieving 30% e ciency. A reason for the discrepancy is not given. French and Widden [5] also established a mathematical relation for the e ciency of the siphon which showed an e ciency of 60% is achievable. Their theoretical model largely suffered from oversimpli cation by assuming a constant water velocity through the siphon and constant drift velocity of air bubbles. The ine ciency was de ned as the ratio of summation of water head loss and buoyancy loss to total available head. This de nition of ine ciency is impractical since it does not relate to the generated power of the air turbine. A similar approach was followed by Howey and Pullen [6] where a slip velocity was
introduced in the calculation of efficiency to account for the velocity difference between water and air bubbles, but still assumed a constant water velocity in the siphon.

The most recent and extensive experimental investigation on a siphonic turbine has been done by Mardiani [7]. He has measured the flow rate of the entrained air for several water discharges and air inlet configurations. He has correctly defined the efficiency as the ratio of the possible generated power by the air turbine to the total input of hydraulic power. However, neither a theoretical model nor an experimental approach that can predict the performance of a siphonic turbine with different hydraulic conditions than his experimental set-up was established.

In the current study, an experimental approach is introduced which can be used to assess the performance of a siphonic turbine with different hydraulic conditions. This approach is based on extensive available experimental data on air transport by an arbitrary geometry and hydraulic conditions. This approach can be divided into:

1. Water head losses and b) air transport head loss, the first and second term in Equation (1) (Pothof and Clemens [10]),

$$\Delta H_{\text{total}} = \Delta H_{\text{w}} + \Delta H_{\text{air}} = \left( \frac{n_i}{D} + \sum \frac{\xi_i}{2g} \right) \frac{v^2}{2g} + \Delta H_{\text{air}}$$

where $\xi_i$ is the head loss coefficient for local losses such as bends and entrance/exit losses and $f$ is the friction factor of the pipeline. $D$, $L$, and $g$ are the pipe diameter, pipe length and earth gravitation constant, respectively. For a turbulent flow with Reynolds number $Re > 4000$ ($Re$ is the proportion of inertia to viscous forces defined as $Re = vD/\nu$ with $v$ as the kinematic viscosity), $f$ is related to the water velocity $v_w$ with the Colebrook-White equation (Idelchik [11]) where $k$ is the wall roughness:

$$\frac{1}{\sqrt{f}} = -2 \log \left( \frac{k/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$

There is not a straightforward formula to determine the air head loss. Pothof and Clemens [10] showed that the air head loss in an inclined pipeline is a function of water and air flow numbers ($F_w$ and $F_a$) defined as:

$$F_w = \frac{Q_w}{A \times (gD)^{0.5}} \quad \text{and} \quad F_a = \frac{Q_a}{A \times (gD)^{0.5}}$$

where $Q_w$ and $Q_a$ are the water and air flow rates and $A$ is the area of siphon pipe cross section. Fig. 2 shows the experimental results for a vertical pipeline where the air head loss is determined for several admitted air flow numbers. For each air flow number, there is a

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe diameter</td>
<td>$D$</td>
<td>3.2 m</td>
</tr>
<tr>
<td>Height of crown</td>
<td>$Z_{\text{crown}}$</td>
<td>$+6.15$ NAP + m</td>
</tr>
<tr>
<td>Slope angle</td>
<td>$\theta$</td>
<td>$45^\circ$ or $90^\circ$</td>
</tr>
<tr>
<td>Height of air inlet</td>
<td>$Z_{\text{inlet}}$</td>
<td>Unknown</td>
</tr>
<tr>
<td>Inlet slope length</td>
<td>$Z_{\text{inlet}}$</td>
<td>Unknown</td>
</tr>
</tbody>
</table>
relation between the air head loss and the water flow number or simply the water velocity according to Equation (4):

\[
\nu_w = F_w \times (gD)^{0.5}
\]

(4)

Using this experimental relation in combination with Equations (1) and (2), the water velocity and the air head loss can be calculated via an iterative numerical process.

### 3.2. Generated power and efficiency of the system

Having air and water discharges, it is possible to calculate the generated power by the air turbine and the efficiency of the system. Details of the performance of the air turbine are not the matter of interest here, and therefore, the following assumptions are made to obtain a simple relation between hydraulic parameters of the system and the performance of the turbine:

- Air is considered as an ideal gas.
- The work is done in the turbine through an isentropic expansion of the air.

Considering these assumptions, the generated power by the turbine is calculated by (Moran and Shapiro [13]):

\[
P_{\text{turbine}} = \frac{m_{\text{air}}}{\eta_i} \left( \gamma \left( \frac{p_{\text{atm}}}{p_{\text{atm}}} \right) \left( \frac{p_{\text{inlet}}}{p_{\text{atm}}} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right)
\]

(5)

where \( p_{\text{atm}} \), \( p_{\text{inlet}} \), \( \gamma \) are the density, pressure, and heat capacity ratio of air at 20°C and atmospheric pressure. \( m_{\text{air}} \) and \( p_{\text{inlet}} \) are the air mass flow rate and pressure at the air inlet of the siphon. \( \eta_i \) is the efficiency coefficient of the air turbine. The system efficiency is defined as the work done by the turbine divided by the total input of hydraulic power:

\[
\eta = \frac{P_{\text{output}}}{P_{\text{input}}}
\]

(6)

It has to be noted that losses in the turbine runner, rotational friction losses and the local losses in the air duct affect the generated power of the air turbine and consequently the system efficiency. Since the performance of the air turbine is not the matter of interest here, these losses are ignored implying that the efficiency of the turbine itself is idealised and set to 100% (i.e. \( \eta_t = 1 \) in equation (5)). Therefore, for an actual turbine with an efficiency of 80%, the generated power and consequently the system efficiency will be 20% lower than the computed value in the paper.

### 4. Approach

The following steps are taken in order to assess the performance of the system:

1. A one dimensional hydrodynamic numerical model of the system is built, taking into account all water losses without considering air entrainment in the system. Using this model, the maximum flow rate through the siphon is computed. This flow rate gives a good indication of the flow regimes which are expected in case of air admission.
2. The water flow rate is reduced step-by-step from the maximum water flow rate computed in the first step. Equation (1) is used to compute the allowable air head loss as the difference between total and water head loss.
3. The experimental data from Figs. 2 and 3 (Kent [12] and Pothof [8]) for slope angles of 45° and 90° are used to determine the air flow number that results in the allowable air head loss.
4. The losses are used to compute the absolute pressure at the air inlet point, from which the generated power and system efficiency are computed (Equations (5) and (6)).

### 4.1. Consistency of experimental results

A direct comparison between the reported values by Pothof [8] and Kent [12] is not possible since the slope angle of 45° was not examined by Pothof [8]. Nevertheless, in order to check the level of consistency between the two experiments, a comparison is made between the results for the slope angles of 45 and 90° obtained by Kent [12] and Pothof [8] respectively. Fig. 3 shows the experimental data of both references. The air head loss is slightly smaller in Kent [12]. This is due to the fact that a larger slope angle results in an easier transport of air in an inclined pipe (Pothof and Clemens [10]). The results show a good consistency with respect to the air head loss as a function of the water flow number.

It has to be noted that for flow numbers lower than 0.7 \( (F_w < 0.7) \), Kent’s data is linearly extrapolated to the point \( F_w = 0.1 \).
and $\Delta H_{\text{air}}/\triangleleft = 1$. This linear behavior is commonly observed for this range of flow numbers ($0.1 < C_w < 0.7$) and slope angles (Pothof and Clemens [10]).

4.2. Uncertainty in using experimental results

An uncertainty may rise due to scaling effects by exploiting the results of experimental studies with relatively small diameters $D < 0.5$ m (Kent [12] and Pothof [8]) for the current system. Pothof and Clemens [10] showed that the effect of the pipe diameter on the clearing flow velocity and consequently the air head loss diminishes above $D = 0.15$ m. In Pothof’s experiment, the pipe diameter is $D = 0.22$ m and therefore no scaling effect is expected. The pipe diameter in Kent’s experiment is $D = 4$ in (102 mm). Therefore, exploiting the Kent’s results for the current system might lead to a rather optimistic results. This should be kept in mind for the results of the siphon with slope angle of 45°.

It has to be noted that in both experimental setups (Kent [12] and Pothof [8]), air has been introduced to the slope via a nozzle installed at the crown of the pipe. Mardiani [7] has shown that the inlet configuration has an influence on the amount of admitted air for a certain available head, or equivalently, on the total head loss for a certain amount of admitted air. However, no quantitative correlation has been provided. In the current study, the details of air inlet are not considered and the results of previous experiments (with a single nozzle) are used.

5. Results

5.1. Step 1: no air admitted

Table 2 shows the results obtained for two configurations of the siphon with no air admitted. The maximum water flow number is $F_w = 0.66$. If air is admitted to the system, this number becomes even less since extra head loss will be introduced by air. Fig. 4 shows the clearing velocity required for several slope angles. Only for small air flow discharges ($F_a < 1000$) the clearing flow number can be reached in the current system. For larger air discharges, a blow-back flow regime with considerable air head losses is expected.

5.2. Step 2: air admitted

5.2.1. Inverted U tube; slope angle of 90°

The attainable water flow rate for several air flow discharges are demonstrated in Fig. 5. By increasing the admitted air, less water is transported through the siphon due to a larger air head loss in the system. As it is seen in the figure, there is a limit to the admitted air. When $F_a = 1000$ reaches 15, the air head loss becomes so large that no water is transported anymore and the siphon breaks. This is shown by a zero water flow rate. Inversely, the generated power and efficiency go up by the increase in the amount of admitted air. More air admission means larger generated power by air turbine and higher efficiency (as shown in Fig. 6). The height of the air inlet also plays a role in the generated power. Higher elevated air inlet leads to a lower local pressure. Therefore, air expands to a lower pressure in the turbine. Consequently, more power is generated and system efficiency improves. Similar to the effect of inlet elevation, tidal condition can affect the performance of the system. At low tide, the system shows a better performance (see Table 3) since the absolute pressure in the system is lower, and therefore, larger differential pressure exists at the air inlet.

It has to be noted that at low pressures ($p < 0.5$ barg) the dissolved air inside the water is released and forms large air pockets which increase the air head loss and hamper the efficiency of the system. This means that the actual efficiency is slightly lower for the situations when the pressure drops below this value. However, quantification of the influence of this phenomenon is rather complicated and is not considered in the current investigation.

The highest efficiency is about 5.7% for a flow number of $F_a = 1000 = 7.5$ (corresponding to $Q_a = 0.43$ m$^3$/s at the air inlet) and an air inlet height of 4.75 NAP. The optimum design conditions with a siphon diameter of 3.2 m are listed in Table 3.

The obtained efficiency shows a large discrepancy with a reported efficiency of 24% by Mardiani [7]. In order to have a better understanding for the reasons of this discrepancy, a quantitative comparison between the results is carried out. Head losses are shown in Fig. 7 where Point A and Point B correspond to highest

Table 2

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>$v_w$ (m/s)</th>
<th>$Q_w$ (m$^3$/s)</th>
<th>$F_w$ (−)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
<td>3.68</td>
<td>37.75</td>
<td>0.66</td>
</tr>
<tr>
<td>90°</td>
<td>3.45</td>
<td>35.38</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Fig. 4. Clearing flow number as a function of admitted air where slopes angles from 5° to 90° are examined (Pothof and Clemens [10]). The dashed line indicates maximum possible water flow number in the current system.

Fig. 5. Water discharge as a function of the height of the air inlet for several admitted air discharges and the low tide condition. $F_a$ is the air flow number at the location of the air inlet.
efficiencies of 5.7% and 24% in the systems. As shown in the figure, the available head in Mardiani [7] is almost twice as that in the current study. This has two implications:

1. The water flow number is significantly greater in Mardiani [7]. Therefore, for an equal air flow number, the air head loss is much smaller. Moreover, for an equal amount of air head loss, much higher air flow numbers can be achieved in Mardiani [7].

2. Due to a larger available head, higher air flow numbers can be reached even if the water flow number is the same. This is simply due to the fact that a larger driving force is available to overcome the head losses.

Consequently, a larger available head results in a larger air–water ratio in Mardiani [7]. This is clearly seen in Fig. 8 where much larger amount of air is transported in Mardiani’s experiment [7]. As it is seen in the figure, at a same water flow number (e.g. \(F_w = 0.8\)), the transported air in Mardiani [7] is significantly larger.

<table>
<thead>
<tr>
<th>Water level condition</th>
<th>Air inlet elevation (NAP + m)</th>
<th>Water flow rate (m³/s)</th>
<th>Air flow rate (m³/s)</th>
<th>Air pocket head loss (m)</th>
<th>Air inlet pressure (bar.a)</th>
<th>Power (kW)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low tide</td>
<td>4.75</td>
<td>23.6</td>
<td>0.43</td>
<td>0.69</td>
<td>0.52</td>
<td>16.4</td>
<td>5.7</td>
</tr>
<tr>
<td>High tide</td>
<td>4.75</td>
<td>23.6</td>
<td>0.43</td>
<td>0.69</td>
<td>0.64</td>
<td>13.4</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Using Equations (5) and (6), the ratio of the efficiencies of the systems is calculated by (superscripts \(S\) and \(M\) stand for the current system and the one studied by in Mardiani [7]):

\[
\frac{\eta^M}{\eta^S} = \frac{m^M/Q_{w}^S}{m^S/Q_{w}^M} \frac{\Delta H_{\text{water}}^M/\phi^M}{\Delta H_{\text{water}}^S/\phi^S}
\]

with

\[
\phi = \left( \frac{\gamma}{\gamma - 1} \right) \left( \frac{P_{\text{atm}}}{P_{\text{atm}} - \rho g h} \right) \left( \frac{1}{\phi^M} - 1 \right)
\]

The contribution of each term in Equation (7) to the efficiency ratio is given in the following:

1. Air-water ratio: \(m^M/Q_{w}^S \approx 0.1 \sim 22.7\)
2. Available head: \(\Delta H_{\text{water}}^M/\phi^M \approx 0.56\)
3. Pressure: \(\frac{P_{\text{atm}}}{\phi^M} \sim 0.33\)

The dominant term which makes a large difference with respect to the efficiency of the systems is the air–water ratio. In Mardiani [7], a larger air–water ratio is achieved, such that air is carried by the water flow in a bubbly flow regime. In the current system, sufficiently high water flow numbers cannot be reached, thus a blow-back flow regime with a large air head loss is expected. Hence less air can be transported which results in a lower efficiency.

5.2.2. Current configuration; slope angle of 45°

The air head loss in a pipe with slope angle of is considerably larger than a vertical pipe (as seen by comparison of Figs. 2 and 3).
For a range of air discharges for which the experimental results are available (i.e. $4.5 < F_a/C_2 < 23.4$), no air can be transported in a siphon with slope angle of $45^\circ$. It might be possible to admit a smaller amount of air in order to run the system. However, this means a frivolous power production and minute efficiency. In order to reduce the air head loss, one solution is to reduce the size of the siphon diameter. By doing so, the water flow number $F_w$ increases, and consequently, the air pockets can be transported with a less head loss. For example, for a smaller diameter of 2 m, a maximum system efficiency of 3.2% is feasible (as demonstrated in Fig. 9).

5.2.3. Optimum design

Comparison of the results for the siphons with slope angle of 45 and $90^\circ$ shows that the larger slope angle leads to a considerably higher efficiency of the system. In order to find the most optimum design with respect to the efficiency, a sensitivity analysis is performed for an inverted U tube siphon where three parameters namely; air discharge, air inlet elevation, and siphon diameter, are adjusted.

![Fig. 8. The air head loss as a function of air flow number. Data points indicated by solid markers are taken from Pothof [8] (also presented in Fig. 2). The empty markers are data from Mardiani’s experiment [7] (also shown in Fig. 7).](image)

![Fig. 9. System efficiency as a function of the air inlet height for several air discharges at low tide. The pipe diameter is $D = 2$ m.](image)

A highest efficiency of 7.2% is observed for $F_a \times 1000 = 15$, $Z_{inlet}=3.62$ NAP + m and $D = 1.3$ m (as seen in Fig. 10). The influence of the air inlet height and the amount of admitted air on the efficiency is already discussed. The change in the diameter of the siphon has two counteracting effects. By decreasing the diameter, $F_w$ increases and hence the air head loss reduces. At the same time, the water head loss in the system increases due to the smaller pipe diameter and Reynolds number which means greater friction factor in the pipeline. Therefore, the water velocity will be reduced through the siphon. Lower velocities result in less air entrainment and consequently less power production. As shown earlier, efficiency is proportional to the amount of admitted air and inverse of water discharge ($\eta \sim m_a/Q_w$). In case of smaller diameter, both nominator and denominator decrease. Hence, the efficiency does not strongly depend on the diameter. It should be noted that a smaller diameter leads to a lower water discharge through a siphon. This means that more number of siphons is required to transport certain amount of water over a dam. For example, to obtain the maximum discharge of 4500 m$^3$/s over the whole Brouwersdam, 1381 siphons with diameter of 1.3 m needs to be installed which is almost 7 times more than the required number of siphons with diameter of 3.2 m.

6. Conclusion and discussions

This paper has presented a new approach to estimate the hydraulic performance of a siphonic turbine. It was shown that siphon diameter, air inlet height and air discharge are the influential parameters on the efficiency of the system. The maximum efficiency was way below the reported values in all previous studies. The reason for this discrepancy was found to be the difference between the laboratory conditions and the hydraulic conditions in an actual low head site. A comparison between the results obtained here and the ones reported by Mardiani [7] showed that the efficiency of the system can be significantly higher at sites with larger available head. Therefore, the usability of siphonic turbine in low-head sites strongly depends on the available head.

Using the described approach, a cost-benefit analysis can be performed to optimise the configuration of the siphon based on the number of siphons versus the generated power and efficiency. Nevertheless, there are several uncertainties in the current study which may lead to deviation of results in a real system and therefore worthy to be investigated further:

- There is no experimental data available for a short length of inclined pipe. A physical model test can be performed to obtain

![Fig. 10. Maximum system efficiency and power as a function of pipe diameter. The air inlet height and amount of admitted air may differ in each point.](image)
air discharge-head loss relations for such a short length. Computational Fluid Dynamics can be also deployed to better understand the air transport phenomenon in the siphon especially the influence of turbulence on bubbles and to reliably calculate the air head loss. Moreover, using CFD analyses, it is relatively easy to assess the siphon performance for various layouts (e.g., different air inlet configurations or installing a vane before the air inlet to introduce a swirl flow in the siphon for preventing the agglomeration of air bubbles into a single air pocket) in order to find the most optimum design.

- The releasing of dissolved air due to negative pressure and consequently formation of air bubbles may have a real hampering effect on the performance of the system. A more complex hydrodynamic model can be developed to consider this effect.
- The generated power and the efficiency of the air turbine depends on the type and the characteristics of the turbine. Since the efficiency of the air turbine directly affects the system efficiency, for a detailed study, these characteristics should be considered.

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References